ACCURACY AND PRECISION OF USNO GPS CARRIER-PHASE TIME TRANSFER

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Abstract

The United States Naval Observatory (USNO) produces GPS carrier-phase time-transfer (GPSCPTT) estimates for approximately 100 GPS-receiver clocks daily. All estimates are available with 16-hr latency; a subset of approximately 34 are available every 6 hours with 3-hour latency plus 24 hours of predictions. All can be downloaded immediately after completion from a USNO Web site. However, despite (or perhaps because of) the continuous nature of this operation, little is known about the precision and accuracy of these estimates.

The goal of this work is to assess the uncertainty of these time transfer values. Comparison measures used include estimates obtained from two-way satellite time/frequency transfer (TWSTFT), and GPS-based estimates obtained from the International GNSS Service (IGS).

USNO GPSCPTT values are estimated to have an uncertainty of several hundred picoseconds when values from the IGS are used as a benchmark in the computation. Frequency values have a few times 10^{-15} fractional frequency uncertainty. TWSTFT values confirm that USNO GPSCPTT estimates are accurate to at least a few nanoseconds.

I. INTRODUCTION

The United States Naval Observatory (USNO) produces GPS carrier-phase time-transfer (GPSCPTT) estimates for approximately 100 geodetic GPS-receiver clocks daily. For each receiver clock, the time transfer estimate is expressed as the time difference of its reading from that of a reference clock at intervals of 5 minutes throughout the day. All USNO GPSCPTT estimates are available with 16-hr latency; a subset of approximately 34 are produced every 6 hours, having 3-hour latency and including 24 hours of predictions. All estimates can be downloaded immediately after completion from a USNO Web site **[1]**.

In this paper, we begin to assess the accuracy/precision of USNO GPS GPSCPTT estimates. The paper includes a summary of the different processing activities at USNO, an assessment of the time and

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14. ABSTRACT
The United States Naval Observatory (USNO) produces GPS carrier-phase time-transfer (GPSCPTT) estimates for approximately 100 receiver clocks daily. All estimates are available with 16-hr latency; a subset of approximately 34 are available every six hours with three hour latency plus 24 hours of predictions. All can be downloaded immediately after completion from a USNO website. However, despite (or perhaps because of) the continuous nature of this operation, little is known about the precision and accuracy of these estimates. The goal of the work whose results will be presented is to assess the accuracy and precision of these estimates, which (may) constitute a vast, un-tapped resource. Comparison measures available include estimates obtained from two-way satellite time/frequency transfer (TWSTFT), BIPM Circular T, and the International GNSS Service (IGS). At present, it is known that averaged across all satellite and receiver clocks in a solution set, USNO post-processed estimates have an RMS of approximately 130 ps with respect to IGS rapid estimates. However, this is after the removal of a system-wide time and frequency offset. The first six hours of USNO clock predictions have an RMS of approximately 2 ns with respect to IGS rapid estimates, after removal of a time and frequency offset.

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frequency uncertainty of USNO GPSCPTT estimates, and conclusions that can be drawn from the experiment.

II. GPS CARRIER-PHASE PROCESSING ACTIVITIES AT USNO

USNO is an analysis center (AC) of the International GNSS Service (IGS) **[2]**. Therefore, four sets of "ultra-rapid" and one set of "rapid" GPS carrier-phase based estimates are computed and submitted to the IGS daily, each set by its own deadline. Each product set includes orbits and time-transfer estimates for the GPS satellites and their clocks, as well as Earth-orientation-parameter estimates such as polar motion and length of day. The "rapid" product set includes time-transfer estimates for the ground-based GPS receiver clocks as well. The IGS uses "rapid" and "ultra-rapid" estimates submitted by each of its ten analysis centers to create its so-called "combined" rapid and ultra-rapid products. Since 2007, the USNO AC has submitted its rapid and ultra-rapid products to the IGS with an on-time rate of 98-99%.

The rapid products are computed every 24 hours and are released with about 16-hour latency. Each set contains 24 hours of post-processed values applicable to the previous day (e.g., a data set released at 16:00 UTC on Day 1 will include values for 00:00 – 23:55 UTC of Day 0). At present, data from approximately 34 GPS receivers are processed together (in "network" mode) in order to compute satellite orbits, Earth-orientation parameters, and GPSCPTT estimates for the satellite clocks and for the clocks of those 34 receivers. Data from approximately 55 additional receivers are processed in precise point positioning (PPP) mode (i.e., analyzed one-by-one for purposes of estimating GPSCPTT and other site-specific values for those receivers) **[3]**.

The ultra-rapid products are computed every 6 hours and are released with three-hour latency. Each set contains 24 hours of post-processed values and 24 hours of predicted values for the same set of parameters as the rapids. At present, data from approximately 34 IGS GPS receivers are processed in network mode to compute the values. Work is underway to increase the number of stations processed both in rapid and ultra-rapid operations.

Figure 1 shows the distribution of the approximately 400 IGS GNSS-receiver tracking stations, a subset of whose data is processed by USNO. Figure 2 provides an overview of the USNO processing structure. The raw GPS data are analyzed using *Bernese 5.0 GPS Software* in combination with a set of in-house-developed automation routines.

Figure 3 shows the distances of the time-transfer links originating at IGS receiver USN3 (located at USNO in Washington, DC) and at SPT0 (located at SP Technical Research Institute of Sweden in Boras, Sweden) produced in USNO GPS processing. Many of the links are several thousand kilometers long. Time-transfer links originating from USN3 have a most-common length of 6000-6999 km. Those originating from SPT0 have 7000-7999 km as the most-common length. We study the uncertainty of USNO GPSCPTT links originating from these two receivers because (a) both are equipped with external hydrogen masers and (b) while USN3 is processed in network mode, SPT0 is processed in PPP mode.

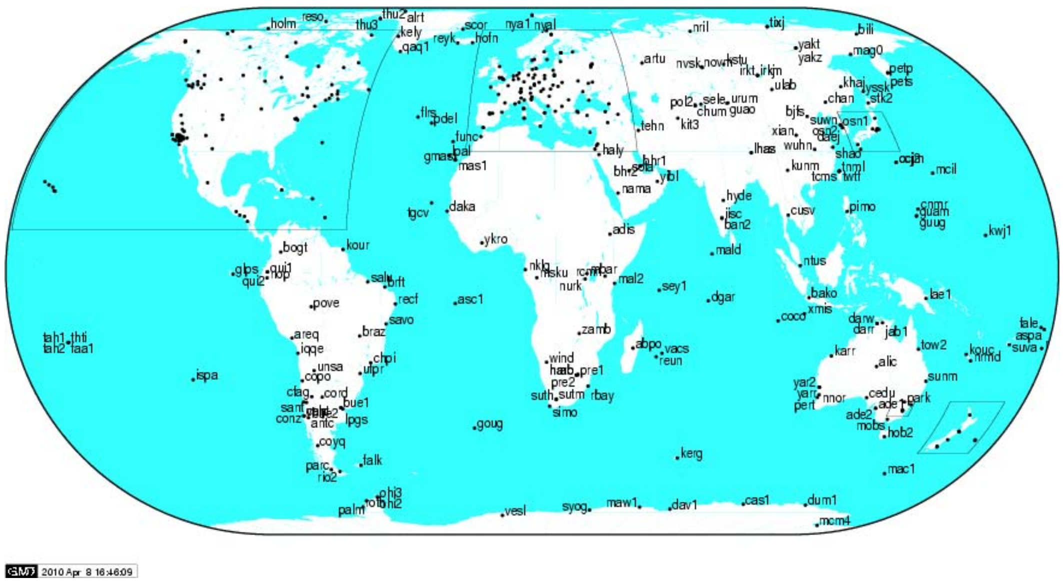


Figure 1. IGS GPS receivers (approximately 400) **[2]**. Data from a subset of these are used to produce USNO rapid and ultra-rapid product sets.

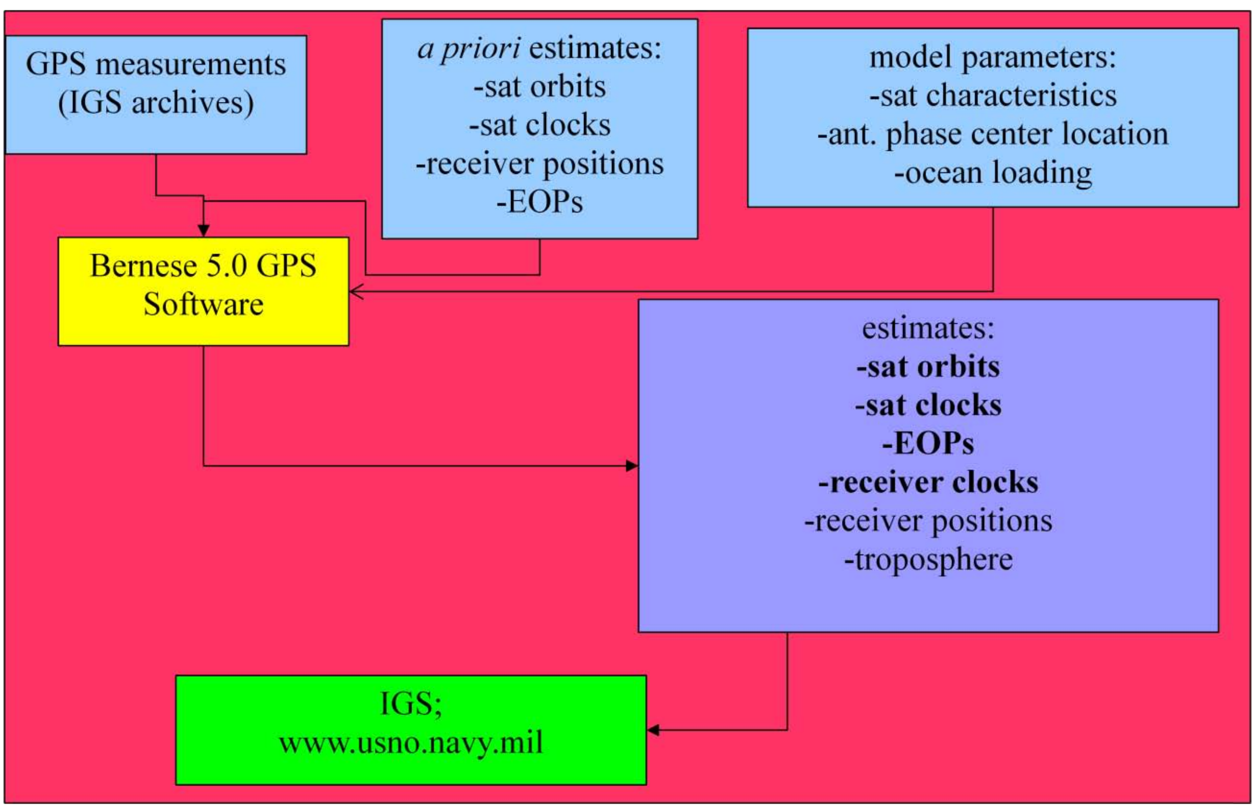


Figure 2. Overview of USNO GPS carrier-phase processing.

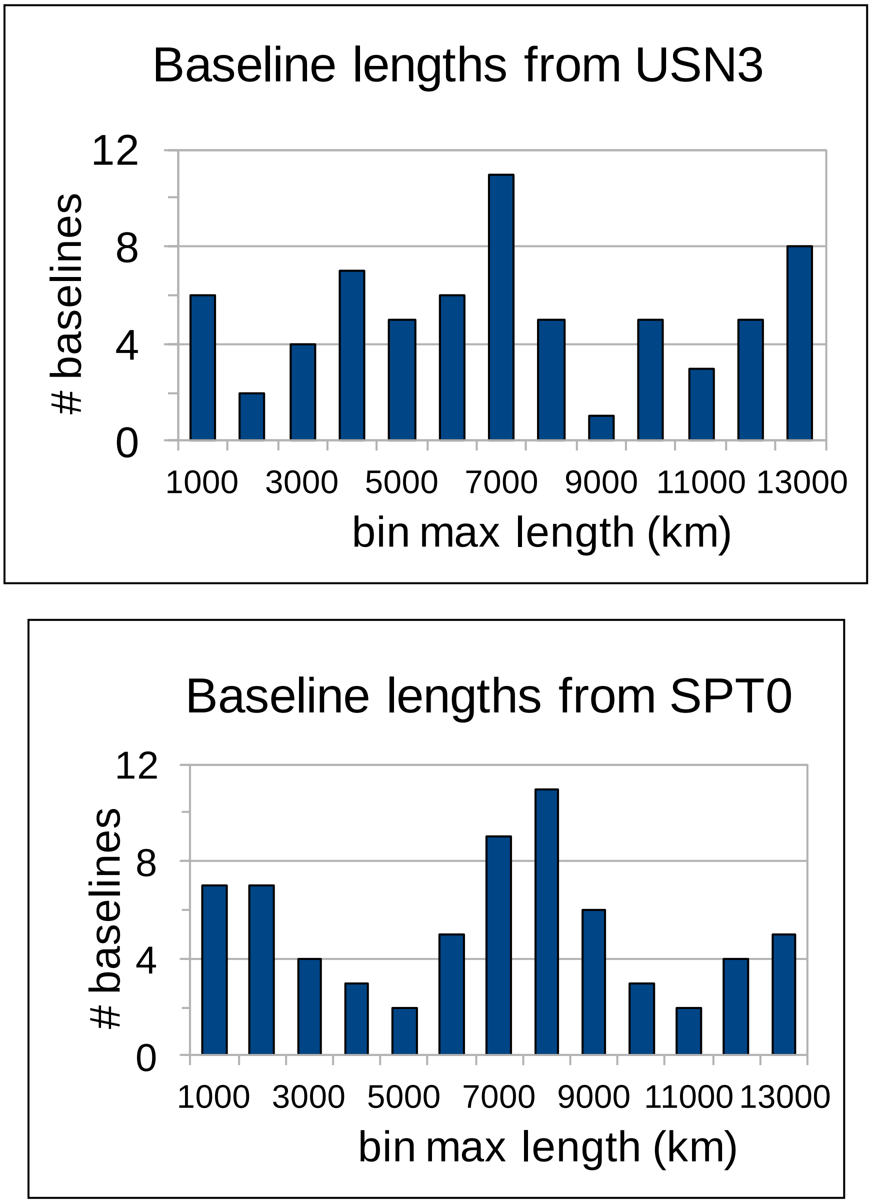


Figure 3. Distribution of time-transfer link distances.

III. METHOD, DATA ANALYZED

The method for assessing the uncertainty of USNO time-transfer estimates is as follows: establish an approximate value for the difference between USNO and IGS time-transfer estimates (described in this section and Section IV), and then apply the uncertainty of IGS time-transfer estimates to obtain an estimate for the uncertainty of the USNO time-transfer estimates.

The uncertainty of USNO rapid time-transfer estimates was assessed using solutions obtained 1 November 2009-29 October 2010. Since predictions are typically the most-used aspect of ultra-rapid

products, the uncertainty of the first 6 hours of USNO ultra-rapid time-transfer predictions was assessed as well, using estimates obtained 16 Oct-1 Nov 2010.

IGS rapid combined time-transfer estimates were used as the primary benchmark in assessing the uncertainty of USNO GPSCPTT estimates. USNO rapid time-transfer estimates were not included in the IGS rapid clock combination during the periods studied, so the IGS combined rapid estimates provide an independent (though GPS-based) benchmark for both the USNO rapid and ultra-rapid estimates. USNO GPSCPTT estimates were compared to those obtained using two-way satellite time/frequency transfer (TWSTFT) along a few timing links as well. This was primarily done as a sanity check: as will be shown in Section IV, diurnals and large excursions in the difference values limited the usefulness of the comparison.

USNO GPSCPTT estimates were compared to the IGS rapid combined time-transfer estimates as follows:

1. **For each day, subtract USNO GPSCPTT estimate of Clock 1 – Clock 2 from IGS combined rapid estimate thereof**. For example, we might subtract the USNO estimate of SPT0-PTBB on MJD 55136 from the IGS combined rapid estimate thereof. Because both USNO and the IGS produce an estimate every 5 minutes, a 24-hr subtraction would yield 288 differences.

2. **Fit a line (mean and slope) to each 24-hour set of differences; compute the standard deviation of the differences from that line**. The mean, slope, and standard deviation provide interesting information:

   The *mean* yields the average time difference between the USNO and IGS solutions. In the SPT0-PTBB MJD 55136 example above, if the mean difference were 10 ps, it would signify that the USNO estimate of SPT0-PTBB is, on average, on that day, 10 ps different than the IGS estimate.

   The *slope* yields the average frequency difference for that day. For example, if the slope computed in the above were 86 ps/d, then the USNO estimate of SPT0-PTBB's relative frequency would be about $1 \cdot 10^{-15}$ different than the IGS estimate.

   The *standard deviation* measures differences between the USNO and IGS estimates not captured above.

3. **Perform the above subtraction and line-fitting for each clock pair for dates 1 Nov 2009-29 October 2010 (rapids) and 16 Oct-1 Nov 2010 (ultra-rapids)**.

4. **For each clock pair, compute the average and standard deviation of the means, slopes, and standard deviations obtained over the analysis period**. Values lying more than five standard deviations from the mean were omitted.

5. **Plot the values obtained in Step 4 according to baseline length**.

The values obtained in steps 1-5 were then used to estimate the overall uncertainty of USNO GPSCPTT estimates, a process described further in the next section.

IV. RESULTS, ESTIMATION OF UNCERTAINTY

IV.A. UNCERTAINTY OF USNO RAPID TIME-TRANSFER ESTIMATES

Figure 4 shows the average mean, slope, and standard deviation for each time link to USNO receiver USN3 with respect to the IGS rapid combined estimate for that link. Figure 4 also shows the standard deviation for each mean, slope, and standard deviation. The statistics for time links between USN3 and each GPS satellite clock, between the GPS 53 satellite clock and each ground receiver clock, and between GPS 53 and the other GPS satellites are shown as well. The statistics of the links to the satellite clocks are included because we suspect that the differences between USNO and IGS clock estimates are due to a difference in the modeling of satellite characteristics, the effects of which are either more or less canceled out in a ground-clock link depending on how much common view of the satellites the ground clocks have (which in turn depends on link distance). The dependence of the statistics on baseline length demonstrated in Figure 4 does not rule out this possibility.

Figure 5 shows the same statistics for time links originating from SPT0. Satellite-clock-based values already shown in Figure 4 are omitted.

We now use the values of Figures 4-5 to estimate the time and frequency difference between USNO GPSCPTT estimates and those of the IGS. We apply a conservative strategy in doing so: our goal is to produce uncertainty estimates that could be used in deciding whether USNO GPSCPTT estimates might meet a given requirement.

Figures 4a-b show that the mean time difference between USNO and IGS time-transfer estimates lies well within ± 25 ps, and that the standard deviation of the time difference between USNO and IGS time-transfer estimates is typically well below 75 ps. We add these in quadrature to obtain an estimate of the uncertainty of the time portion of the difference between USNO and IGS time-transfer estimates: $\sigma_{\text{time, USNO-IGS}} = 79$ ps. Note: since we have combined systematic (25 ps) and random (75 ps) errors, the combined estimate of 79 ps would not necessarily decrease as $\sqrt{N}$ (where N = number of measurements) were the number of measurements increased. Only the random (75 ps) portion of the uncertainty would.

Figures 4c-d show that the mean slope difference between USNO and IGS time-transfer estimates lies within ± 125 ps/d, and that the standard deviation of the slope difference between USNO and IGS time transfer estimates is typically well below 250 ps/d. Adding in quadrature yields a value for $\sigma_{\text{freq, USNO-IGS}}$, the uncertainty of the frequency of USNO time-transfer estimates with respect to those of the IGS, of 280 ps/d.

Figure 4e shows the average standard deviation of the residuals remaining after a mean and slope are removed from the difference between USNO and IGS time-transfer estimates. These typically lie below 80 ps. Figure 4f shows that the scatter in these values also typically lies below 80 ps. Adding in quadrature yields a value for $\sigma_{\text{noise, USNO-IGS}}$, the uncertainty of USNO time-transfer estimates with respect to those of the IGS due to noise, of 113 ps.

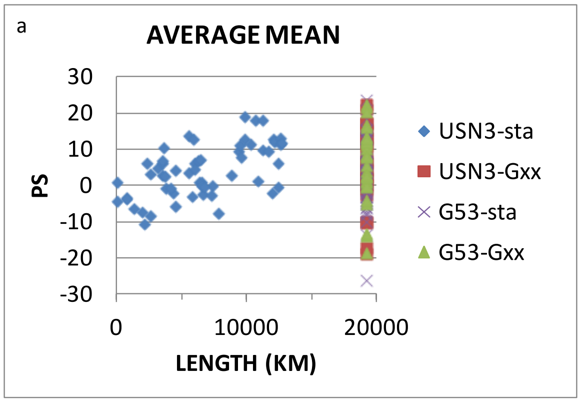


Fig 4a. Average time differences between USNO and IGS rapid time-transfer estimates for time links originating at USN3.

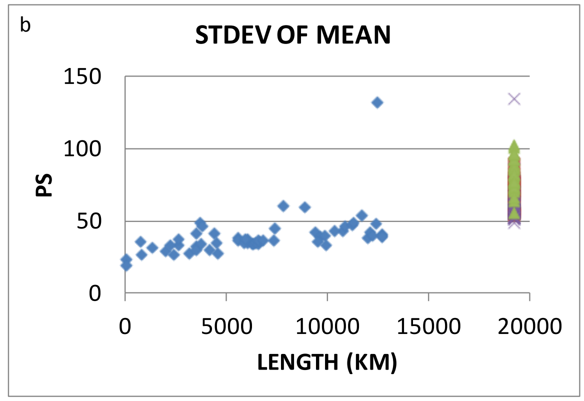


Fig 4b. Standard deviation of the time differences between USNO and IGS rapid time-transfer estimates for time links originating at USN3.

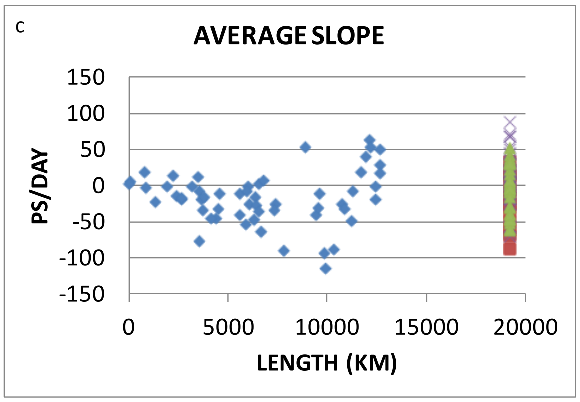


Fig 4c. Average frequency differences between USNO and IGS rapid time-transfer estimates for time links originating at USN3.

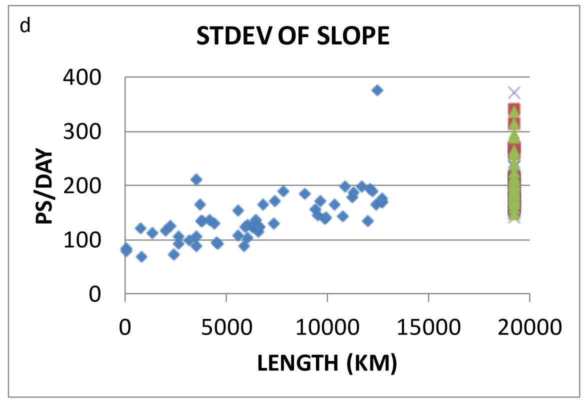


Fig 4d. Standard deviation of the frequency differences between USNO and IGS rapid time-transfer estimates for time links originating at USN3.

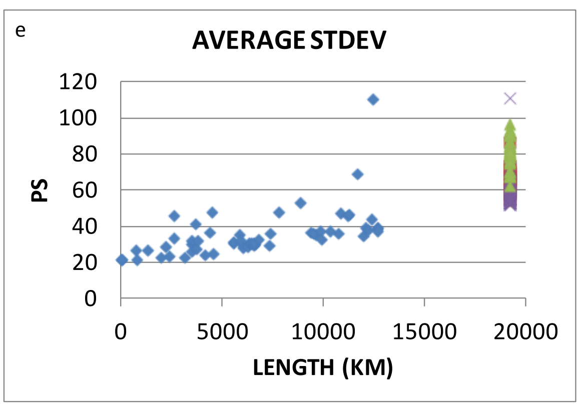


Fig 4e. Average standard deviation of linear fit between USNO and IGS rapid time-transfer estimates for time links originating at USN3.

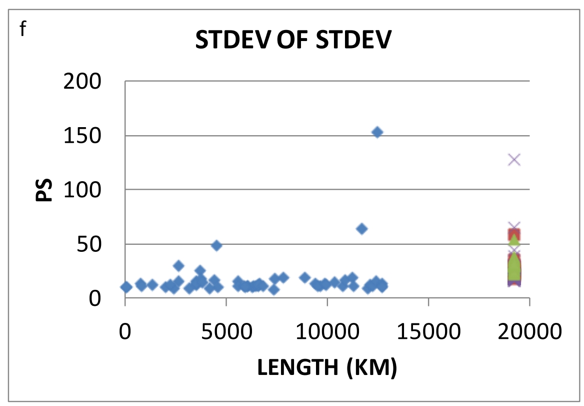


Fig 4f. Standard deviation of standard deviation of linear fit between USNO and IGS rapid time-transfer estimates for time links originating at USN3.

Figure 4. Statistics of differences between USNO time-transfer estimates and IGS rapid time-transfer estimates for links originating at GNSS receiver USN3 (Washington, DC). Each blue diamond represents a time-transfer link between two ground-based receivers. Red, purple, and green markers represent links to or between clocks on GPS satellites.

Repeating the calculation for time links originating at SPT0 (Figure 5), we see that the mean time difference between USNO and IGS time-transfer estimates lies well within ± 40 ps (Figure 5a), and that the standard deviation of the time difference between USNO and IGS time transfer estimates is below 150 ps (Figure 5b), yielding $\sigma_{\text{time, USNO-IGS}}$ = 155 ps. The mean slope difference between USNO and IGS time transfer estimates lies within ± 250 ps/d (Figure 5c), and the standard deviation of the slope difference between the time-transfer estimates is typically 500 ps/d (Figure 5d), yielding $\sigma_{\text{freq, USNO-IGS}}$ of 559 ps/d. The average standard deviation of the residuals remaining after a mean and slope are removed from the difference is well below 120 ps (Figure 5e), and the scatter of these values is typically less than 100 ps (Figure 5f), yielding $\sigma_{\text{noise, USNO-IGS}}$ of 156 ps.

The uncertainty of the time-difference values produced by USNO GPSCPTT at an averaging time (τ) of 1 day can then be estimated as follows:

$$\text{USNO Time Uncertainty } (\tau = 1\text{ d}) = [\sigma^2_{\text{time, USNO-IGS}} + \sigma^2_{\text{freq, USNO-IGS}} \cdot (1\text{ d})^2 + \sigma^2_{\text{noise, USNO-IGS}} + \sigma^2_{\text{IGS}}]^{1/2} \quad (1)$$

where σ_{IGS} denotes the uncertainty of IGS rapid time-transfer estimates. Because the IGS estimates that its clock values have an RMS of approximately 75 ps with respect to the IGS timescale **[4]**, we estimate that $\sigma_{\text{IGS}} = [2\cdot(75\text{ ps})^2]^{1/2}$ = 106 ps.

Using Equation 1 and the values derived above, we see that GPSCPTT estimates originating from USN3 (Figure 4) have a time uncertainty of 329 ps ($\sigma_{\text{time, USNO-IGS}}$ = 79 ps, $\sigma_{\text{freq, USNO-IGS}}$= 280 ps/d, $\sigma_{\text{noise, USNO-IGS}}$= 113 ps, σ_{IGS} =106 ps). Estimates originating from SPT0 (Figure 5) have a time uncertainty of 610 ps (155 ps, 559 ps/d, 156 ps, 106 ps). The standard deviation of the mean slope differences between USNO and IGS estimates (Figures 4d and 5d) are the largest contributors to the uncertainty.

Above, we estimated the uncertainty of time-difference values obtained using USNO rapid GPSCPTT. We can estimate the uncertainty of frequency-difference values obtained from GPSCPTT as follows:

$$\text{USNO Frequency Uncertainty } (\tau = 1\text{ d}) = [\sigma^2_{\text{freq, USNO-IGS}} + 2\cdot\sigma^2_{\text{noise, USNO-IGS}} + 2\cdot\sigma^2_{\text{IGS}}]^{1/2} \quad (2)$$

Using the values derived from Figures 4-5, we estimate that the USN3-centered links have a frequency uncertainty of approximately 355 ps/d ($4\cdot10^{-15}$) ($\sigma_{\text{freq, USNO-IGS}}$= 280 ps/d, $\sigma_{\text{noise, USNO-IGS}}$= 113 ps, σ_{IGS} = 106 ps). Those originating from SPT0 have an uncertainty of approximately 619 ps/d ($6\cdot10^{-15}$) (559 ps/d, 156 ps, 106 ps). The standard deviation in the mean slope differences between USNO and IGS estimates (Figures 4d and 5d) are again the largest contributors to the uncertainty.

Another way to study the difference between USNO GPSCPTT and IGS time-transfer estimates is to simply subtract (Clk 1 – Clk 2) $_{\text{USNO}}$ from (Clk 1 – Clk 2) $_{\text{IGS}}$ over a long period of time and then compute Allan and time deviations of the differences. Figure 6 shows the result of doing this for five baselines (ranging in distance from 623 to 7352 km) over the period 1 Nov 2009-29 Oct 2010. As Figure 6 shows, the frequency stability of the USNO-IGS difference is about $1\cdot10^{-15}$ at τ = 1 d, the time stability is 37-70 ps at 1 d, and both types of stability tend to get worse with increased baseline length.

Figure 7 shows the difference between USNO GPSCPTT estimates and those obtained from TWSTFT for four baselines. These are unfiltered values with only a mean removed; only simple attempts were made to remove time steps in the TWSTFT estimates caused by equipment changes. The goal of performing this subtraction is to provide an independent estimate of the uncertainty of the GPSCPTT estimates. (While the comparison to the IGS is valid, there could be systematic errors affecting both.)

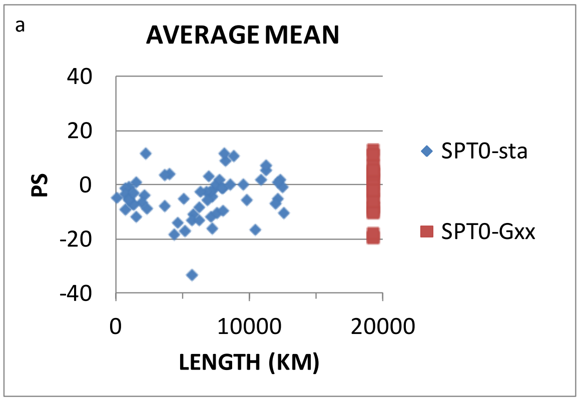


Fig 5a. Average time differences between USNO and IGS rapid time-transfer estimates for time links originating at SPT0. Each dot represents one time-transfer link.

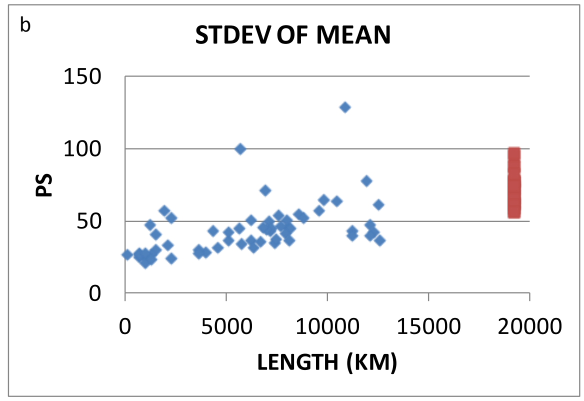


Fig 5b. Standard deviation of the time differences between USNO and IGS rapid time-transfer estimates for time links originating at SPT0.

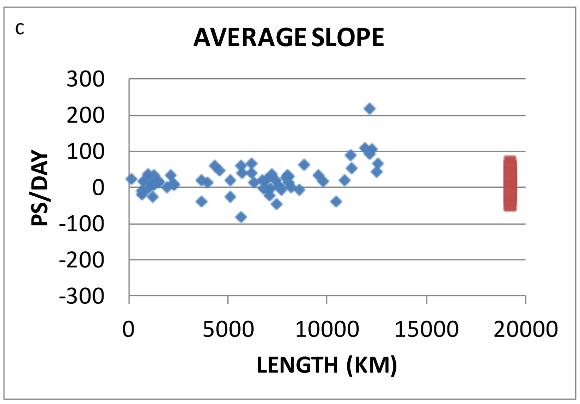


Fig 5c. Average frequency differences between USNO and IGS rapid time-transfer estimates for time links originating at SPT0.

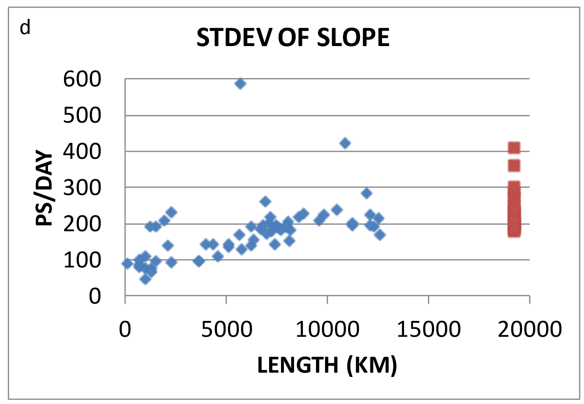


Fig 5d. Standard deviation of the frequency differences between USNO and IGS rapid time-transfer estimates for time links originating at SPT0.

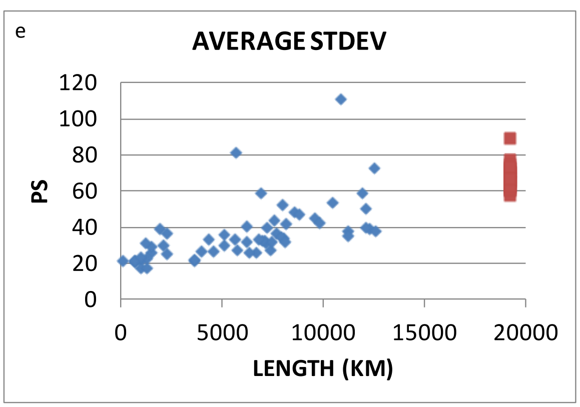


Fig 5e. Average standard deviation of linear fit between USNO and IGS rapid time-transfer estimates for time links originating at SPT0.

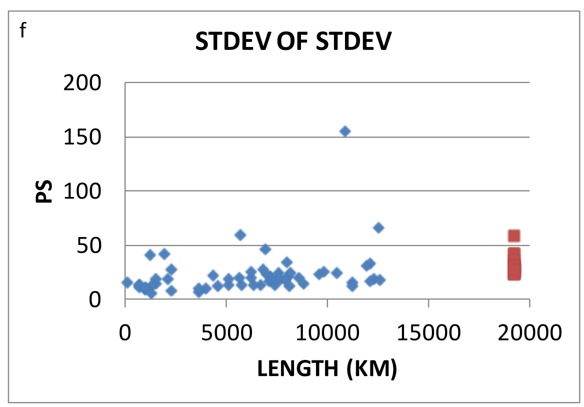
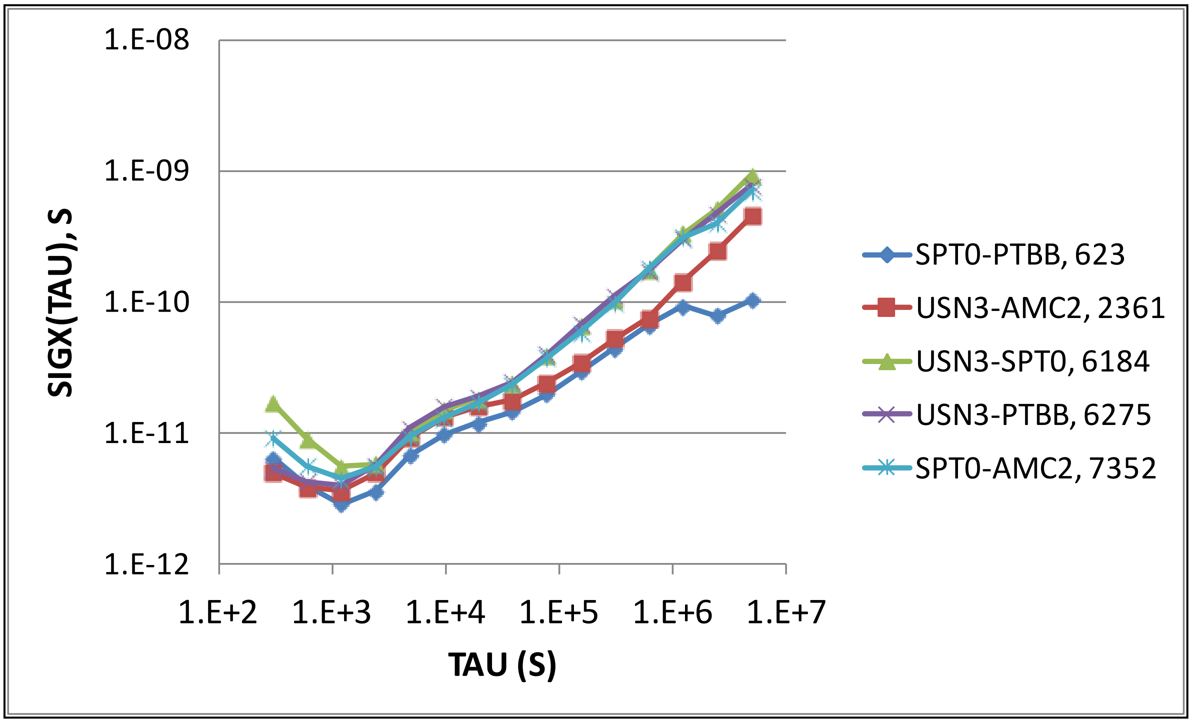


Fig 5f. Standard deviation of standard deviation of linear fit between USNO and IGS rapid time-transfer estimates for time links originating at SPT0.

Figure 5. Statistics of differences between USNO time-transfer estimates and IGS rapid time-transfer estimates for links originating at GNSS receiver SPT0 (Boras, Sweden). Each blue diamond represents a time-transfer link between two ground-based receivers. Red squares represent links from SPT0 to satellite clocks.



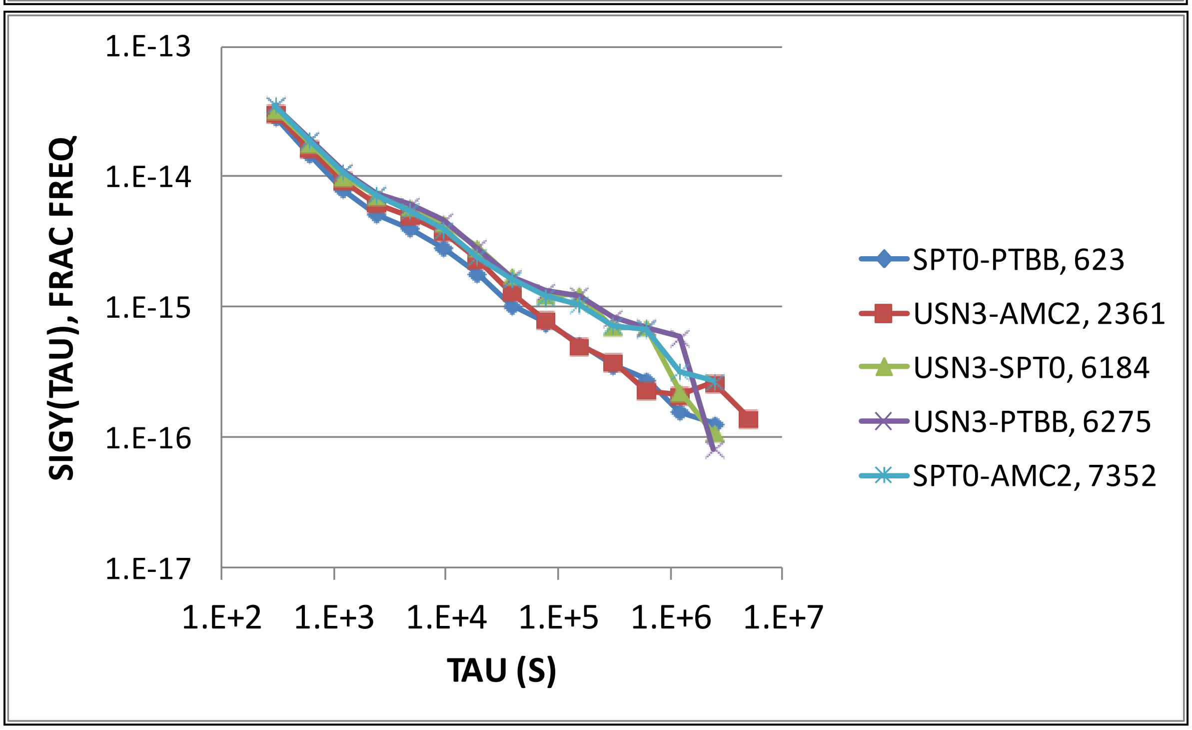


Figure 6. Allan and time deviation of (Clk 1 – Clk 2)$_{\text{USNO rapid}}$ – (Clk 1 – Clk 2)$_{\text{IGS rapid}}$ 1 November 2009-29 October 2010. The distance of each link is given in kilometers. PTBB is located in Braunschweig, Germany; AMC2 is located in Colorado Springs, Colorado.

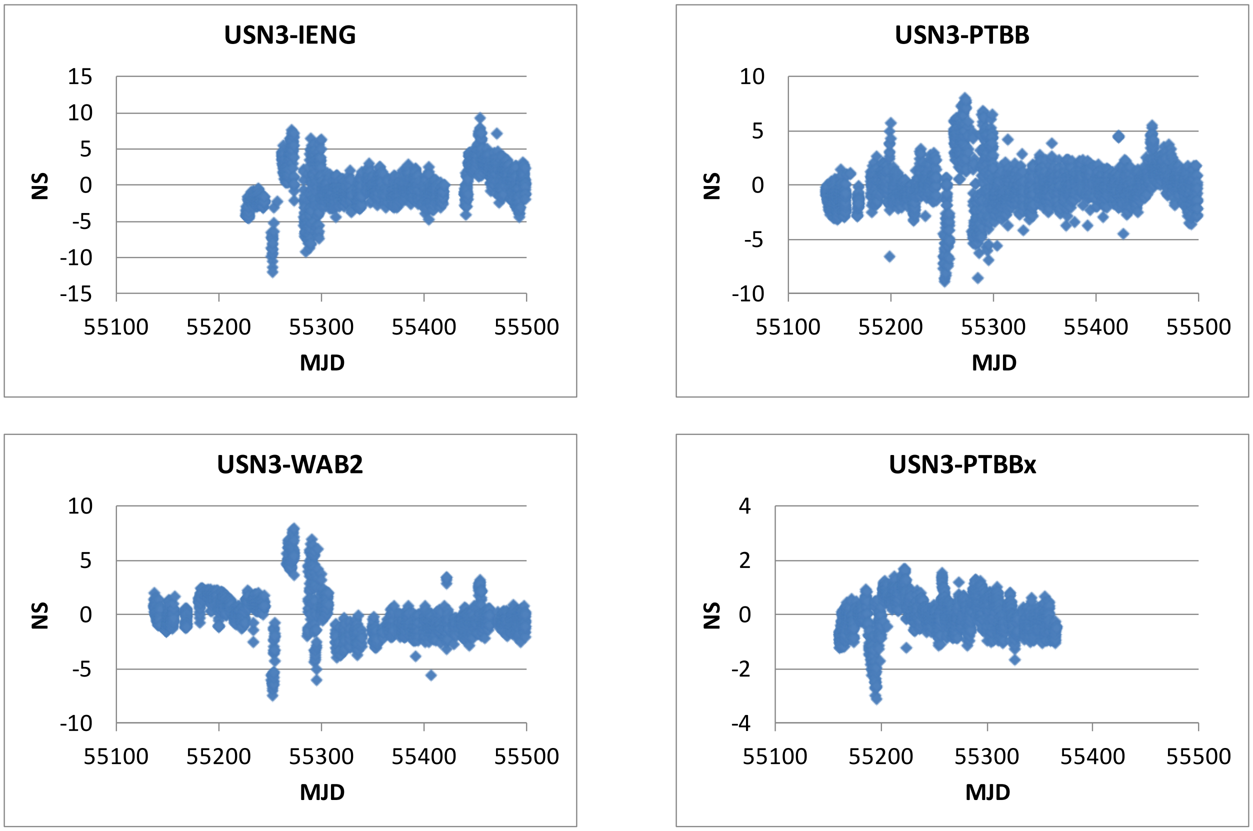


Figure 7. Difference between USNO GPSCPTT estimates and those obtained from TWSTFT. A mean value was removed from each series, as were discontinuities associated with equipment changes. All TWSTFT comparisons were performed using Ku-band measurements, with the exception of USN3-PTBBx, which was performed using X-band. IENG is located in Torino, Italy, WAB2 in Wabern, Switzerland, and PTBB in Braunschweig, Germany.

Recall that the time uncertainty derived from the comparison to the IGS values yielded uncertainties of 300-700 ps. Figure 7 confirms that the USNO rapid GPSCPTT estimates are at least accurate to several nanoseconds.

IV.B. Uncertainty of First 6 Hours of USNO Ultra-rapid Time-Transfer Predictions

Figure 8, constructed similarly to Figures 4-5, allows us to estimate the uncertainty of the first 6 hours of USNO ultra-rapid time-transfer predictions. The mean time difference between USNO GPSCPTT and IGS time-transfer estimates lies well within ±2.5 ns (Figure 8a) and the standard deviation of the time difference between USNO and IGS time-transfer estimates is below 10 ns (Figure 8b), yielding $\sigma_{\text{time, USNO-IGS}}$ = 10 ns. The mean slope difference between USNO and IGS time-transfer estimates lies within ± 15 ns/d (Figure 8c), and the standard deviation of the slope difference between the time-transfer estimates is better than 60 ns/d (Figure 8d), yielding $\sigma_{\text{freq, USNO-IGS}}$ of 62 ns/d. The average standard deviation of the residuals remaining after a mean and slope are removed from the difference between USNO and IGS time-transfer estimates is below 10 ns (Figure 8e), and the scatter of these values is

typically less than 6 ns (Figure 8f), yielding $\sigma_{noise,\ USNO\text{-}IGS}$ of 12 ns. Using these values, we estimate that USNO ultra-rapid time-transfer predictions have a time and frequency uncertainty at 1 d of 64 ns and 64 ns/d, respectively.

These uncertainties – again driven by the standard deviation of the slope – seem large compared to the, say, 10 ns one might expect given the values in the IGS ultra-rapid processing summaries **[5]**. Further investigation is needed due to the sparseness of the data. A refinement of the estimation method is likely in order as well.

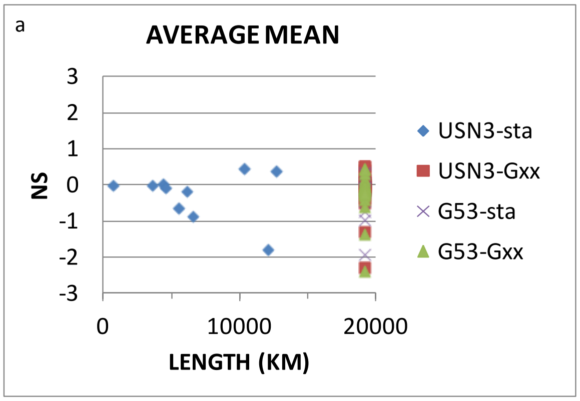


Fig 8a. Average time differences between USNO ultra-rapid and IGS rapid time-transfer estimates for time links originating at USN3.

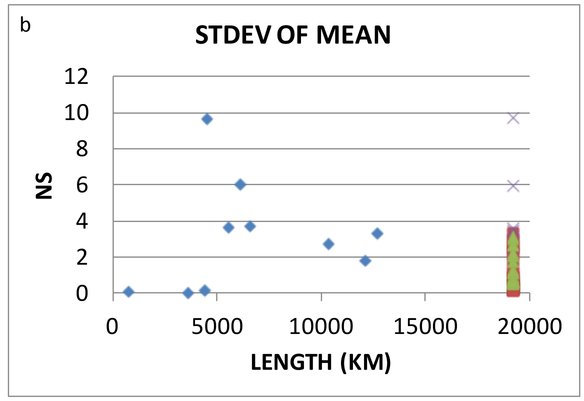


Fig 8b. Standard deviation of the time differences between USNO ultra-rapid and IGS rapid time-transfer estimates for time links originating at USN3.

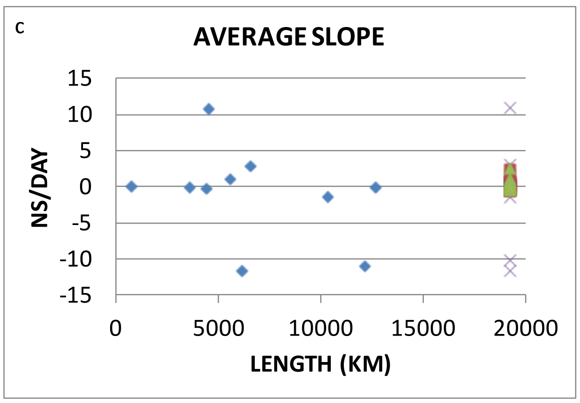


Fig 8c. Average frequency differences between USNO ultra-rapid and IGS rapid time-transfer estimates for time links originating at USN3.

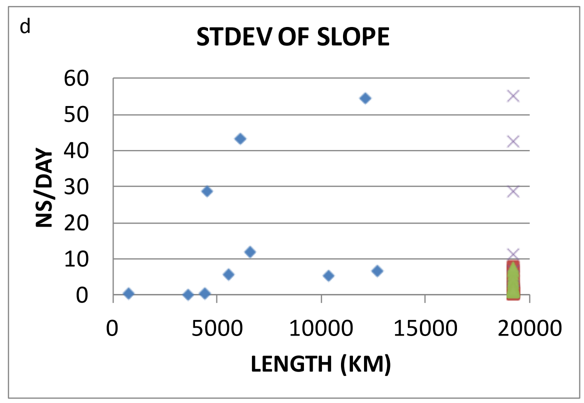


Fig 8d. Standard deviation of the frequency differences between USNO ultra-rapid and IGS rapid time-transfer estimates for time links originating at USN3.

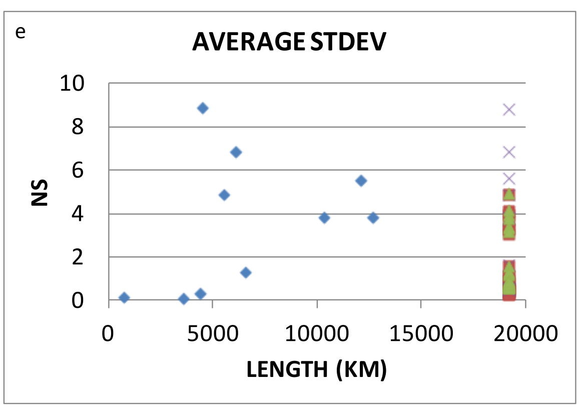


Fig 8e. Average standard deviation of linear fit between USNO ultra-rapid and IGS rapid time-transfer estimates for time links originating at USN3.

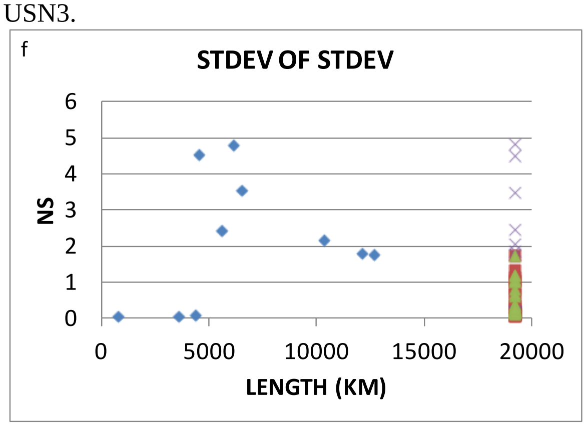


Fig 8f. Standard deviation of standard deviation of linear fit between USNO ultra-rapid and IGS rapid time-transfer estimates for time links originating at USN3.

Figure 8. Statistics of differences between USNO ultra-rapid predicted time-transfer estimates (first 6 h) and IGS rapid time-transfer estimates for links originating at GNSS receiver USN3 (Washington, DC). Each blue diamond represents a time-transfer link between two ground-based receivers. Red, purple, and green markers represent links to or between clocks on GPS satellites.

V. DISCUSSION

The noise of the GPSCPTT-TWSTFT differences (several ns) is considerably larger than the uncertainty of the USNO GPSCPTT estimates as computed using the IGS rapid clock combination as a benchmark (a few hundred picoseconds). As Figure 9 shows, there is a near-diurnal periodic component in the GPSCPTT-TWSTFT differences with amplitude of several ns which appears to cause much of the noise.

We do not yet know whether the source of this periodicity is TWSTFT or GPSCPTT. Both TWSTFT and GPSCPTT are known to exhibit temperature-dependent hardware delays, which would cause diurnal errors, though the problem is thought to be more pronounced in TWSTFT **[6]**. Furthermore, GPS satellites repeat their geometry (with respect to the ground) every 23 h 56 min; geosynchronous satellites used in TWSTFT have diurnal position shifts as well. Searching both sets of time-transfer estimates for near-24-hour periodic signals could help determine whether the several-ns difference between the two techniques should be assigned to the GPSCPTT error budget.

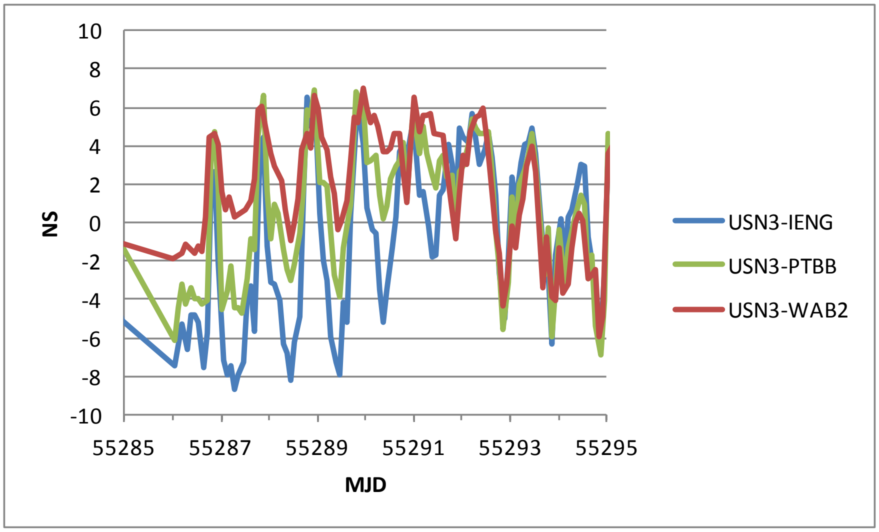
The largest portion of the uncertainty values assigned to USNO time- and frequency-transfer estimates originates from the standard deviation of the frequency differences between USNO and IGS rapid solutions. Comparing Figure 4c to 4d and Figure 5c to 5d reveals that, while on average, the frequency difference has both positive and negative signs, resulting in a small average value, the frequency difference between the USNO and IGS solutions on a given link on a given day can be large.

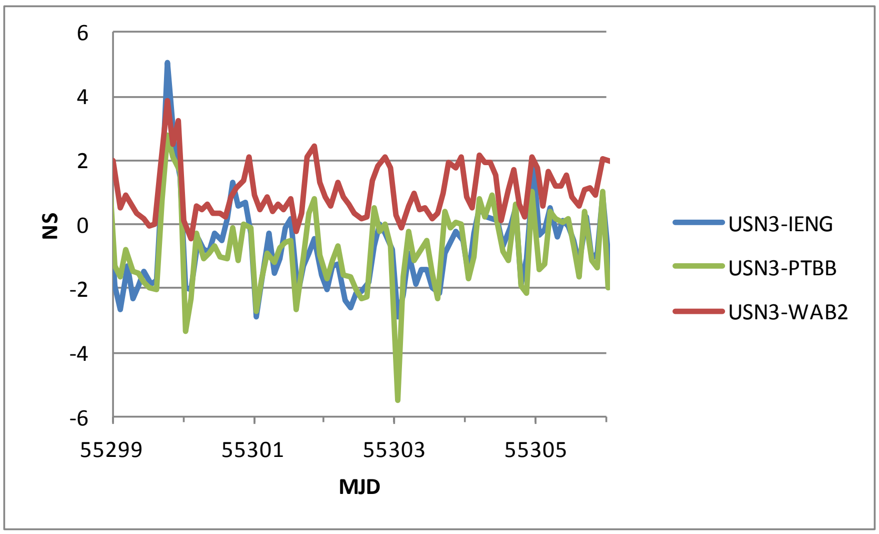
At least two factors could contribute to this. First, the set of GPS receiver data used to compute a solution varies from day to day, both at USNO and at the IGS. When network processing is used, the addition or subtraction of a GPS receiver's data affects the solution, if for no other reason that it impacts ambiguity resolution. This, in turn, affects the accuracy of GPSCPTT frequency estimation **[7]**. Unfortunately, USNO at present uses only approximately 34 stations in its network solution, which makes solution sensitivity to stations chosen worse.

Second, in the network solution for receiver and satellite time-transfer estimates, no reference clock is chosen. Rather, the sum of the receiver time-transfer estimates is fixed to zero. This is unphysical; however, an attempt to switch to using a real reference clock resulted in very noisy time-transfer estimates. (Interestingly, the above test appeared to improve the quality of USNO network-solution-derived Earth-orientation parameters **[8]**, which have been problematic **[9,10]**). The above time-transfer strategy could lead to frequency errors that vary from day to day. If frequency accuracy is the desired goal, then a different strategy may be needed.

Figures 4 and 5 showed that time links originating from USN3 had lower uncertainty than those originating from SPT0. The reason for this is unclear. However, USN3 data are processed in the network part of the solution, and SPT0 data are processed in the PPP portion. It would be interesting to see if, in USNO processing, network-solution time-transfer estimates are generally less noisy than PPP time-transfer estimates. One could begin by refining the baselines shown in Figures 4-5 so that only network-network links are shown in Figure 4 (USN3) and only PPP-PPP links are shown in Figure 5 (SPT0).

The Allan deviation and time deviation values for τ = 1 d ($1\cdot10^{-15}$ or better, 37-70 ps) shown in Figure 6 are considerably smaller than the frequency and time uncertainty values ($4\text{-}6\cdot10^{-15}$, 300-600 ps) derived from Figures 4-5. Part of this is because the Allan/time deviation values do not include the uncertainty of the IGS time-transfer estimates (~106 ps in time, 106 ps/d = $1.2\cdot10^{-15}$ in frequency). To investigate the





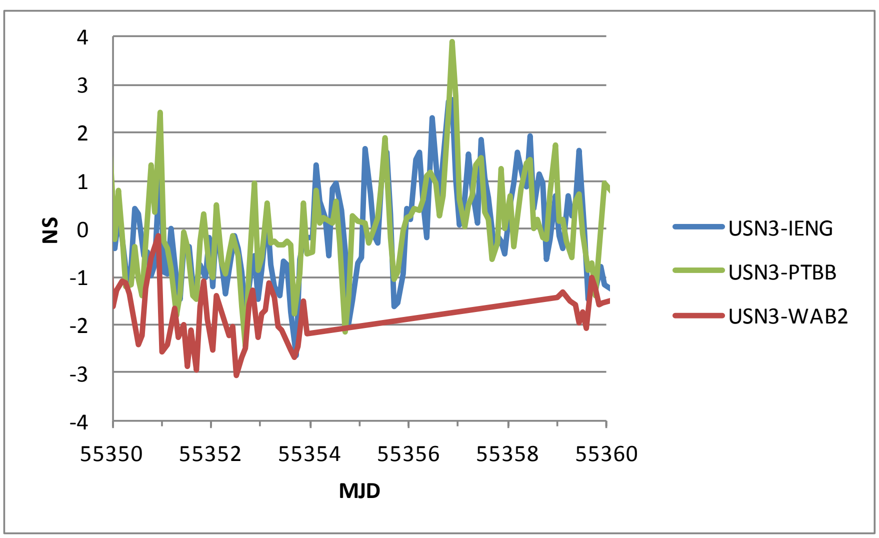


Figure 9. Expanded views of the GPSCPTT-TWSTFT differences shown in Figure 7. Diurnal signals are evident.

remainder of the difference, we pulled out the statistics shown in Figures 4-5 for the individual links shown in Figure 6 and calculated time/frequency uncertainty according to Equations 1 and 2, excluding the IGS-clock contributions thereto. As Table 1 shows, the uncertainties calculated this way range from 95 to 153 ps and $1.6–2.4\cdot10^{-15}$. This closes part of the gap.

Table 1. Comparison of uncertainty values obtained using Equations 1-2 (minus IGS uncertainty contribution) to Allan/time deviation values shown in Figure 6.

link	$\sigma_{\text{time, USNO-IGS}}$	$\sigma_{\text{freq, USNO-IGS}}$	$\sigma_{\text{noise, USNO-IGS}}$	**time uncertainty (1 d)**	$\sigma_x(\tau = 0.89$ d)	**freq uncert. (1 d)**	σ_y ($\tau = 0.89$ d)
	ps	ps/d	ps	ps	ps	ps/d (ff, 10^{-15})	fractional frequency
SPT0-PTBB	25	88	25	95	19	137 (1.6)	$7.6\cdot10^{-16}$
USN3-AMC2	28	76	26	85	24	127 (1.5)	$7.9\cdot10^{-16}$
USN3-PTBB	35	133	31	141	40	195 (2.2)	$1.3\cdot10^{-15}$
SPT0-AMC2	36	146	31	153	37	207 (2.4)	$1.2\cdot10^{-15}$

VI. CONCLUSIONS

Traceable to the IGS rapid timescale:

1. At $\tau = 1$ d, USNO rapid GPSCPTT estimates have a few hundred picoseconds time uncertainty and a few hundred ps/d ($\sim 4\text{-}6\cdot10^{-15}$) frequency uncertainty. USNO rapid GPSCPTT estimates also have a 30-70 ps time stability and $\sim 1\cdot10^{-15}$ frequency stability relative to the IGS at 1 d.
2. USNO ultra-rapid time-transfer predictions have several tens of ns time uncertainty at 1 d, and several tens of ns/d frequency uncertainty at 1 d. More study is needed to refine these estimates.

Comparisons to TWSTFT confirm that the USNO rapid GPSCPTT estimates are accurate to several ns.

The above estimates are conservative, and no efforts were made to improve the uncertainty of any particular baseline. Improved uncertainty along a given link could likely be achieved simply by focusing on its characteristics.

VII. ACKNOWLEDGMENTS

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